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Flow of a dusty gas through the annular space between two concentric circular cylinders

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In this paper the problem of flow of a dusty gas through the annular space between two concentric circular cylinders has been considered. The gas is assumed to contain uniform distribution of dust. The flow is produced by the motion of cylinders. Two cases viz., (i) cylinders moving exponentially with time, and (ii) cylinders moving in simple harmonic motion have been considered.

Introduction

In the formulation of Saffman (1962) the motion of a dusty gas is represented in terms of a large number density $N(x, t)$ of very small particles, and it is assumed that the bulk concentration of the particles is small enough to be neglected. On the other hand the density of the material of dust is assumed to be large compared to gas density, so that f, the mass concentration of dust is an appreciable fraction of unity. Also assuming that dust particles are small enough to make the Stokes' law of resistance between the particles and the gas appropriate, the equations of motion become

$$\frac{\partial \bar{u}}{\partial t}+(\bar{u}.\nabla)\bar{u} = -\frac{1}{\rho} \text{ grad } p+\nu\nabla^2\bar{u}+ \frac{\kappa N}{\rho}(\bar{v}-\bar{u}), \qquad \dots \quad (1)$$

$$m\left(\frac{\partial \bar{v}}{\partial t}+(\bar{v}.\nabla\bar{v})\right) = \kappa(\bar{u}-\bar{v}), \qquad \dots \quad (2)$$

$$\text{div } \bar{u} = 0, \qquad \dots \quad (3)$$

$$\frac{\partial N}{\partial t}+\text{div}(N\bar{v}) = 0, \qquad \dots \quad (4)$$

where $\bar{u}$, $\bar{v}$ are the gas and dust velocities, m the mass of a dust particle, κ the Stokes resistance coefficient which for spherical particles is $6\pi\mu a$ and μ, ρ, p and ν have their usual meanings.

Solution of Problem

In the present problem we assume that $N = N_0$, a constant throughout the motion. Since the velocities are independent of z, and there are no velocities in r and θ indirections, $\bar{u}$ and $\bar{v}$ are $\bar{u} = \{0, 0, u(r, t)\}$ and $\bar{v} = \{0, 0, v(r, t)\}$. Equations (3) and (4) are satisfied. We have to solve equations (1) and (2). These equations reduce to

$$\frac{\partial u}{\partial t} = \nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\frac{\kappa N_0}{\rho}(v-u); \qquad \ldots (5)$$

and

$$\tau\frac{\partial v}{\partial t} = u-v, \qquad \ldots (6)$$

where $\tau = m/r$ is the relaxation time of the dust and $\partial p/\partial z$ has been taken equal to zero.

With the help of dimensionless time variable t/τ and dimensionless length $r/(\nu t)^{\frac{1}{2}}$, these equations become

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+f(v-u) \qquad \ldots (7)$$

and

$$\frac{\partial v}{\partial t} = u-v \qquad \ldots (8)$$

where t and r are now dimensionless and $f = \dfrac{N_0 m}{\rho}$ is the mass concentration of dust.

Let

$$\left.\begin{aligned} u &= u(r)\exp(-\alpha t),\\ v &= v(r)\exp(-\alpha t).\end{aligned}\right\} \qquad \ldots (9)$$

Taking the radius of inner cylinder as unit of length, let b be the radius of the outer cylinder. The boundary conditions are

$$\left.\begin{aligned} u &= w_1\exp(-\alpha t), \quad r = 1,\\ u &= w_2\exp(-\alpha t), \quad r = b.\end{aligned}\right\} \qquad \ldots (10)$$

Substituting (9) in equations (7) and (8), we have

$$\frac{d^2 u}{dr^2}+\frac{1}{r}\frac{du}{dr}+\lambda^2 u = 0, \qquad \ldots (11)$$

$$v = \frac{u}{1-\alpha}, \qquad \ldots (12)$$

where $\lambda^2 = \left(1+\frac{f}{1-\alpha}\right)$ and u and v here stand for $u(r)$ and $v(r)$. Solving equation (11), and using boundary conditions (10) we have

$$w(r) = \frac{w_1 T_0(r, b, \lambda)+w_2 T_0(1, r, \lambda)}{T_0(1, b, \lambda)} \tag{13}$$

where

$$T_0(a, b, \lambda) = J_0(\lambda a)\, Y_0(\lambda b)-J_0(\lambda b)\, Y_0(\lambda a).$$

Hence the gas velocity and dust velocity are

$$u = \exp(-\alpha t)\, \frac{w_1 T_0(r, b, \lambda)+w_2 T_0(1, r, \lambda)}{T_0(1, b, \lambda)} \tag{14}$$

and

$$v = \exp(-\alpha t)\, \frac{w_1 T_0(r, b, \lambda)+w_2 T_0(1, r, \lambda)}{(1-\alpha)T_0(1, b, \lambda)} \tag{15}$$

The results of calculation are incorporated in tables 1 and 2. Table 1 shows the change in dust velocity and table 2 shows the variation of velocities in a dusty and non-dusty gas for different values of $r(1 \leqslant r \leqslant 2)$.

Table 1. Dust velocity : $f = 0{\cdot}2$, $t = 1{\cdot}0$, $\alpha = 0{\cdot}5$

r \ w_2/w_1	1	$\frac{1}{2}$	2
1 0	1.213	1.213	1.213
1.2	1.292	1.114	1 650
1.4	1 326	1.001	1.977
1.5	1.329	0.941	2.108
1.6	1.322	0.877	2 211
1.8	1.282	0 744	2.358
2.0	1.213	0 607	2.426

Let

$$u = u(r)\exp(i\Omega t),$$

$$v = v(r)\exp(i\Omega t).$$

Substituting these values in equations (7) and (8) we have

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}+(\alpha+i\beta)^2 u = 0, \quad \ldots (16)$$

Table 2. Velocities of a dusty and non-dustry gas : $t = 1{\cdot}0$, $\alpha = 0{\cdot}5$

	$f = 0{\cdot}0$			$f = 0{\cdot}2$		
r \ w_2/w_1	$\frac{1}{2}$	1	2	$\frac{1}{2}$	1	2
1.0	.607	.607	.607	.607	.607	.607
1.2	.548	.635	.809	.557	.646	.825
1.4	.488	.646	.963	.501	.663	.989
1.5	.458	.647	1.026	.470	.665	1.054
1.6	.427	.644	1.079	.439	.661	1.106
1.8	.365	.631	1.161	.372	.641	1.179
2 0	.303	.607	1 213	.303	.607	1.213

where

$$(\alpha+i\beta)^2 = -\frac{f\Omega^2+i\Omega(1+f+\Omega^2)}{1+\Omega^2}.$$

The boundary conditions are

$$\left.\begin{aligned} u &= w_1 \exp(i\Omega t), & r &= 1, \\ u &= w_2 \exp(i\Omega t), & r &= b. \end{aligned}\right\} \tag{17}$$

Solution of equation (16) under the boundary conditions (17) is

$$u = \frac{w_1 T_0(r, b, \alpha+i\beta)+w_2 T_0(1, r, \alpha+i\beta)}{T_0(1, b, \alpha+i\beta)}.$$

Hence the gas and dust velocities are

$$u = \exp(i\Omega t)\frac{w_1 T_0(r, b, \alpha+i\beta)+w_2 T_0(1, r, \alpha+i\beta)}{T_0(1, b, \alpha+i\beta)} \tag{18}$$

and

$$v = \frac{\exp(i\Omega t)}{1+i\Omega}\,\frac{w_1 T_0(r, b, \alpha+i\beta)+w_2 T_0(1, r, \alpha+i\beta)}{T_0(1, b, \alpha+i\beta)}. \tag{19}$$

Results of numerical calculations (NBS 1947, 1950) are incorporated in tables 3 and 4. Table 3 shows the change in dust velocity and table 4 shows the variation of velocities in a dusty and non-dusty gas for different values of $r(1 \leqslant r \leqslant 2)$.

Table 3. Dust Velocity; $f = 0{\cdot}2$, $\Omega = 1{\cdot}0$

w^2/w_1	$t = 0{\cdot}0$			$t = 1{\cdot}0$		
1.0	.5000	.5000	.5000	.6909	.6909	.6909
1.2	.3907	.4423	.5457	.5986	.6901	.8729
1.4	.3187	.4176	.6155	.5206	.6894	1.0270
1.5	.2938	.4161	.6609	.4862	.6897	1.0966
1.6	.2752	.4213	.7134	.4543	.6901	1.1617
1.8	.2541	.4498	.8412	.3967	.6906	1.2796
2 0	.2500	.5000	1.0000	.3455	.6909	1.3818

Table 4. Velocities of a dusty and non-dusty gas; $w_2/w_1 = 1{\cdot}0$, $\Omega = 1{\cdot}0$

$t =$	0·0		0·5		1·0	
r \ f	0.0	0.2	0.0	0.2	0.0	0.2
1.0	1.0000	1.0000	0.8775	0.8775	0.5403	0.5403
1.2	0.9014	0.9784	0.9114	0.9035	0.6085	0.6075
1.4	0.9871	0.9687	0.9251	0.9141	0.6366	0.6357
1.5	0.9869	0.9685	0.9259	0.9152	0.6383	0.6379
1.6	0.9878	0.9709	0.9231	0.9135	0.6325	0.6322
1.8	0.9928	0.9822	0.9073	0.9015	0.5998	0.5996
2.0	1.0000	1.0000	0.8775	0.8775	0.5403	0.5403

Conclusion

Tables 1 and 2 show the values of velocities for the case when the cylinders move exponentially with time. It can be seen from table 2 that the dusty gas moves faster than the non-dusty gas for the values considered. It is also clear that the dust velocity is greater than the gas velocity. Tables 3 and 4 give the values of velocities when the cylinders execute simple harmonic motion. For the values considered, it is clear from the tables that the dust velocity is less than the gas velocities and the non-dusty gas travels faster than the dusty gas. We thus conclude that in the first case the dust increases the velocity of the gas while in the second it decreases the gas velocity.

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